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The Use of Circular Scales in the Perspective Grid Technique of Making Photographic Measurements

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ABSTRACT: The perspective grid technique commonly utilizes a square reference scale in the camera's field of view from which a grid pattern is graphically laid out in the photograph to measure distances lying in the same object plane as the scale. An improved technique described in this paper uses three circular scales as a basis for determining the X and Y coordinates of a point in the object plane from the x and y coordinate positions of its image in the photograph. This method offers the advantages of better accuracy and less tedious graphic methods of grid construction. A step-by-step procedure of constructing rectangular coordinate systems using either circular or square reference scales is presented.

KEYWORDS: engineering, photography, scale (ratio)

The interpretation of analytical photographs recorded for scientific, engineering, and forensic science objectives may be conveniently divided into two distinctly different categories: *qualitative* analysis and *quantitative* analysis. Photographs made simply for qualitative purposes fall into a class of photography commonly referred to as documentary recording. Photographs intended for quantitative measurement, on the other hand, can usually provide the analyst with specific metric information regarding sizes, distances, locations, orientations, and other dimensional data that may be needed for complete detailed analyses of the objects and scenes recorded. Frequently, the only characteristic that separates these two classes of photographs is the inclusion of a suitable metric scale of reference at the proper location in the camera's field of view.

This paper treats the specific case relating to the quantitative analysis or mapping of features that fall in a single plane in the object field which is recorded obliquely by the camera. Traffic accident scene recording is a specific application of these techniques, where tire marks, accident debris, and other artifacts of interest to the investigator are distributed over a reasonably flat roadway surface. Crime scene recording of objects distributed across the floor of a room is also exemplary of the methods to be described. In these and other cases, the objective is to produce a plan view in the form of a scale drawing or map showing the arrangement of artifacts across the surface of the roadway, floor, or other flat surface.

The graphical or mathematical solutions to these photogrammetric problems require known locations of either (1) four control points in the object plane or (2) three control points and the focal distance between camera lens and image. The determination of the latter value may be subject to uncertainty when the lens is not focused at infinity or when lenses of the zoom type are employed. The control points must be visible in the photograph and no more

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than two can lie along a single straight line. Linear scales or rulers commonly included in photographs are inadequate for the purpose of rectifying oblique images because all of the points on the scale fall along a straight line. At least one more point at a known location relative to the linear scale is required if the focal distance is also known; otherwise, two additional control points are needed. The perspective grid technique described by Baker [1] fulfills the first condition; that is, it provides four points of known location that all fall in the object plane where the measurements are to be performed. These points are defined by the four corners of a square scale of known size placed in the same object plane where the measurements are to be made (Fig. 1). The perspective grid method permits measurements to be

FIG. 1—*Perspective grid technique applied to automobile accident scene measurements. (Photo courtesy of Jack Whitnall.)*

made by either simple geometrical construction using ordinary drafting tools or by more rigorous mathematical techniques.

Conventional Techniques

The perspective grid can be thought of as one square element in a grid structure from which the other square grid elements can be generated. When viewed from an oblique angle, the square perspective grid scale is not imaged as a square but as a trapezoid. When two opposite edges are extended by lines drawn away from the position of the camera, they converge to a point referred to as the *vanishing point*. These lines are called *orthogonals*. If two or more squares are placed side by side, mutually parallel edges of all squares converge toward the same vanishing point. Equidistant transversal lines in the object plane are defined by the fore and aft sides of the trapezoidal image. The spacings between them diminish in proportion to the convergence of the orthogonals. This is the fundamental principle that makes it possible to plot an entire grid system from only one rectangular element. It is important to point out that the edges of any other squares that might not be aligned parallel with the above converge toward other unique vanishing points, all of which fall along a common straight line referred to as the *vanishing line*. The geometrical process of gridding a photograph from a rectangular perspective grid scale is described in detail by Baker [*1*]. The reader is referred to this source for further information on this graphic process. Applications in accident scene photography are discussed by Whitnall [*2*].

Expected error in determining the true positions of grid lines by the geometrical construction process outlined in the above reference is about $\pm 2\%$ for distances in the object plane less than ten times the overall dimensions of the perspective grid scale. For example, the expected error in locating a point 10 m away from a perspective grid scale 1 m^2 in size is about ± 20 cm using good drafting techniques. This is about the best accuracy that can be achieved by a skilled draftsman with the patience needed to meticulously carry out the graphic construction process. Errors tend to be cumulative in the construction process because the position of each horizontal grid line depends upon the location of the line earlier laid down next to it.

A major source of errors is the location of vanishing points and the vanishing line connecting them. These errors arise from attempting to extend lines along the edges of the perspective grid scale over distances on the photograph that are large compared to the size of the scale itself. In Fig. 1, for example, the construction lines in locating the vanishing point are greater than 4½ times the length of the scale edges from which they were extended. A slight misalignment with the edge of the scale results in a significant error in defining the vanishing point. The errors in locating all grid lines are highly dependent upon the accuracy to which the vanishing points can be defined. Further errors are introduced when attempting to interpolate graphically between grid lines to locate the positions of unknown points in the object plane.

Circular Scale Method

Hyzer first suggested the use of circular reference scales in 1981 [*3*], which has led to methods that overcome several of the inaccuracies associated with conventional graphic methods of analysis using square scales. When viewed or photographed from an oblique angle, circles are not imaged as circles, but as ellipses. Referring to Fig. 2, the angles of obliquity (θ) measured from center lines drawn perpendicular to the plane in which the circles lie are related to the major and minor axes of the ellipses as follows:

$$\cos\theta = \frac{\text{minor axis}}{\text{major axis}} \tag{1}$$

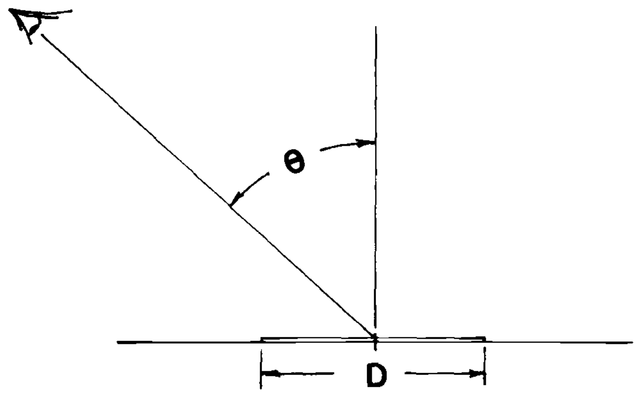

FIG. 2—*A circle viewed from an oblique angle (θ) greater than zero appears elliptical in shape.*

Only if the image and object planes are strictly parallel will circular scales be imaged as true circles. The angles of obliquity from two or more circular scales in the same object plane define the location of the camera lens relative to that plane, except for the unique case of parallel object and image planes. Vanishing points may be determined from two or more circular scales of the same diameter in a photograph by drawing tangent lines between two circular scales as first suggested by Hyzer [*4-6*]. Two circles of equal diameter at different distances from the camera are required to locate a single vanishing point. Two vanishing points and a vanishing line may be established by means of three circular scales of equal diameter at separated locations in the camera's field of view (Fig. 3).

Circular scales are best arranged in the picture area by placing one in foreground center and the other two in the left and right background to bracket the area of interest fore and aft

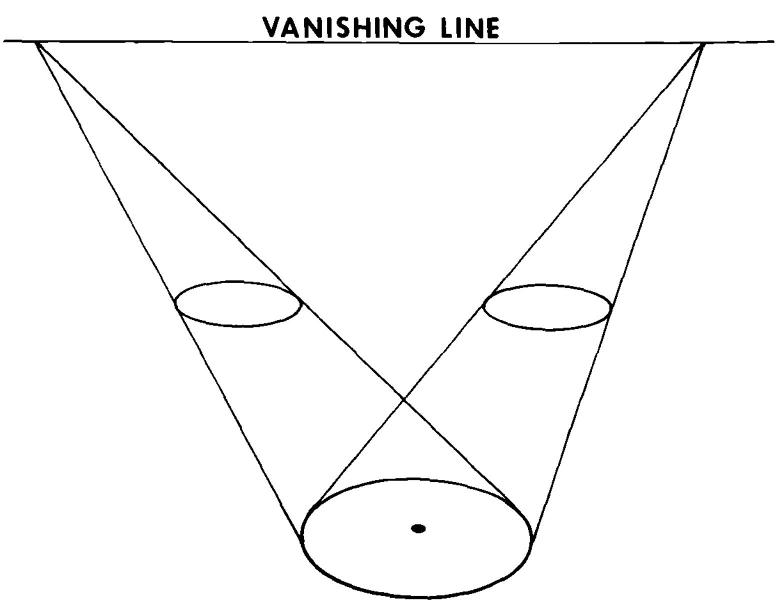

FIG. 3—*Locating the vanishing line from three circular scales of identical size in the same plane.*

and left and right. The center point of the foreground circle should be visible in the photograph. It is not necessary to know either the actual or relative positions of the circular scales in the object field to establish vanishing points and vanishing lines. A determination of the relative sizes of the major axes in the elliptically shaped images may be utilized to determine the relative distances between the camera and the circular scales in the object plane. Since circular scales can usually be widely separated in the plane where the measurements are to be made, the accuracy of defining vanishing points and vanishing lines is significantly improved over that obtainable with square scales.

With the vanishing line drawn in, a coordinate system is next laid out graphically onto the photographic print as follows:

1. With reference to Fig. 4, draw two tangent lines to the largest ellipse that run parallel to the vanishing line. Extend these lines entirely across the photograph. The line farthest from the vanishing line is taken as the x axis in the photograph, which corresponds to the X axis or zero reference line in the object plane.
2. Drop a perpendicular from the vanishing line at k through the center Point c of the circle forming the above ellipse to intersect the x axis. This defines the origin o. Line ok is taken as the y axis in the photograph, which corresponds to the Y axis in the object plane.
3. Extend two lines from Point k through their tangency points on the above ellipse. The trapezoidal figure mnqp inscribing the ellipse is the first element in the grid structure from which the others may be constructed.

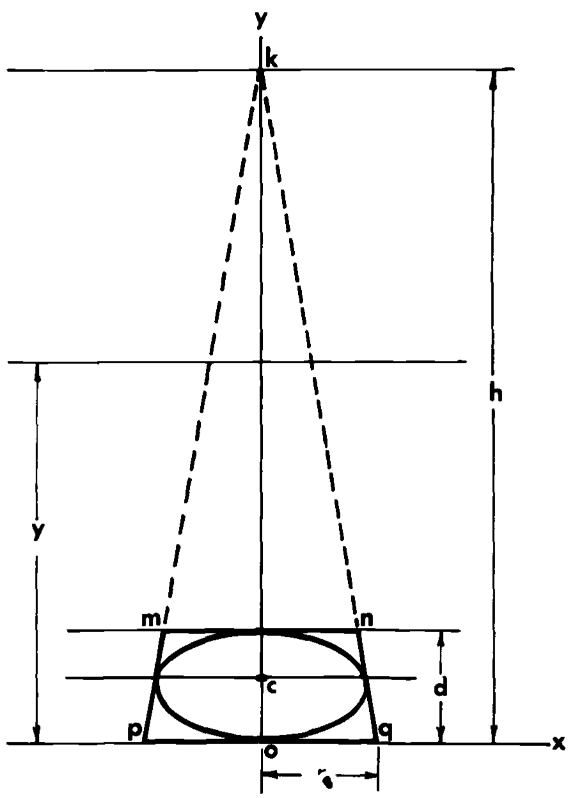

FIG. 4—*Constructing an* x, y *coordinate system in the photograph.*

4. Measure the distance r along the x axis.

5. Now, referring to Fig. 5, the X and Y coordinate positions of any point in the object plane can be computed from the x and y coordinates of its recorded image by means of the following equations:

$$S = D\left[\frac{h}{d} - 1\right] \tag{2}$$

$$Y = S\left[\frac{h}{h - y} - 1\right] \tag{3}$$

$$X = \frac{DX}{r}\left[\frac{h}{h - y}\right] \tag{4}$$

where

D = diameter of a circular scale or edge length of a square scale, m;
S = distance in the object plane between the zero reference line tangent to the near side of the circular or square scale and a perpendicular dropped from the camera lens, m;
d = minor axis of the ellipse or the distance between the parallel sides of the trapezoidal figure in the image, arbitrary units;
h = image distance between the x axis and the vanishing line, same arbitrary units as above;
x and y = coordinates of a point in the image, arbitrary units; and
X and Y = coordinates corresponding to the above in the object plane, m.

6. To construct grid lines in the photograph, mark off distances equal to $2r$ along the left and right extensions of the x axis.

7. Draw lines between the above marked-off points and Point k on the horizon line.

8. Referring to Fig. 4, compute the image distance (y) between the x axis and each successive horizontal grid line by means of the following equation:

$$y = h\left(1 - \frac{S}{S + Y}\right) \tag{5}$$

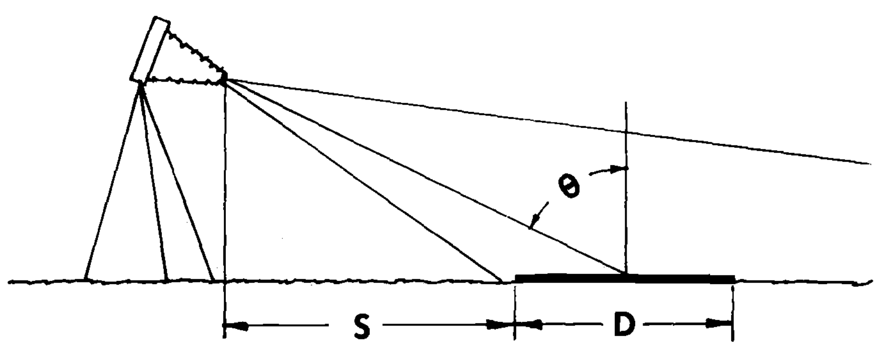

FIG. 5—*Elevation view of camera and reference scale.*

where

y = y coordinates of the grid lines in the photograph, same arbitrary units as above and
Y = Y coordinates of the grid lines in the object plane, m.

To plot equidistant transversal grid lines in the photograph, a series of calculations are performed to compute the distances y in the print which correspond to object distances Y. Typical values of Y are D, $2D$, $3D$, $4D$, $5D$, and so forth up to $10D$. Measurements in the image can be made in any convenient and consistent set of units, such as millimetres, centimetres, inches, and so forth. Equidistant transversals are then laid out at the computed distances from the x axis to provide a complete grid system, as shown in Fig. 1.

Conclusions

There are at least five advantages to using circular scales in place of square perspective grid scales and computing the coordinate positions as outlined above. The advantages are:

1. Reduced error in defining the location of the vanishing line, which results in increased accuracies in specifying coordinate positions and grid line locations.
2. Computations of coordinate positions and grid line locations can be performed quickly and accurately in a programmable calculator thereby speeding up the grid process and minimizing the chances of making mistakes.
3. Cumulative errors inherent in the pure graphical approach are eliminated by calculating the positions of each grid line independently.
4. The X and Y coordinates of any point in the object plane can be easily determined from the corresponding x and y coordinates of that same point in the image, thereby eliminating the need to interpolate distances between grid lines.
5. Transversal grid lines in the photograph are always parallel to the vanishing line. Transversal grid lines constructed from a square perspective grid scale are parallel only when the film plane in the camera is aligned strictly parallel with the zero reference line in the object field.

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